Field Oriented Control in Permanent Magnet Synchronous Motors

Dr. Antoni Arias.
Universitat Politècnica de Catalunya.
Catalonia

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• Electrical Motors Classification
• Permanent Magnet Synchronous Machine Model
• Field Oriented Control
  – Torque, speed and position control
• Sensorless Field Oriented Control
Classification

• Types of electrical machines
  – DC
  – Universal
  – AC
    • Single-phase AC Induction Motors
    • Single-phase AC Synchronous Motors
    • Three-phase AC Induction Motors
    • Three-phase AC Synchronous Motors
  – Stepper
  – Permanent Magnet
    • PMDC – Brushless
    • PMAC – SMPM, IPM
  – Linear
  – Nano

Most important and used
Permanent Magnet Machines

- Magnetic material to establish the rotor flux.
  - Most common magnetic material are samarium-cobalt (SmCo) and neodymium-iron-boron (NdFeB) introduced in 1983 having superior magnetic characteristics at room temperature.

- Advantages:
  - No rotor currents => no rotor losses.
  - Higher efficiency => energy saving capability.
  - Smaller rotor diameters, higher power density and lower rotor inertia.
  - Higher torque per ampere constant.
  - Weight and volume less than other type of machine for the same power. Attractive for aerospace applications such as aircraft actuators.
  - Other applications: machine tools, position servomotors (replacing the DC motors).

- Inconvenience:
  - synchronous machines => need for rotor position.
  - Price
PM Motor Types

• Considering the shape of the back EMF
  • PMDC – brushless - trapezoidal
  • PMAC – sinusoidal
    – Surface Mount PM
    – Interior Mount PM

SMPM  IPM
PMSM Dynamic Equations

- **Space vector transformation**
  - combines the individual phase quantities into a single vector in the complex plane
  \[
  i_{\alpha\beta} = \frac{2}{3} \left( i_a(t) + i_b(t) e^{\frac{j 2\pi}{3}} + i_c(t) e^{\frac{j 4\pi}{3}} \right)
  \]
  - \( i_a = i_a(t) \); \( i_\beta = \frac{\sqrt{3}}{3} \left( i_b(t) \quad i_c(t) \right) \)
  - Similar transformation are applied to
    - Stator voltages
    - Stator flux linkage
PMSM Dynamic Equations

- Basic equation for phase windings voltages

\[
\begin{bmatrix}
v_a \\
v_b \\
v_c 
\end{bmatrix} = \begin{bmatrix}
i_a \\
i_b \\
i_c 
\end{bmatrix} \frac{d}{dt} \begin{bmatrix}
\psi_a \\
\psi_b \\
\psi_c 
\end{bmatrix} + r_s
\]

- Total flux linkage

\[
\begin{bmatrix}
\psi_a \\
\psi_b \\
\psi_c 
\end{bmatrix} = \begin{bmatrix}
L_a & M_{ab} & M_{ac} \\
M_{ba} & L_b & M_{bc} \\
M_{ca} & M_{cb} & L_c 
\end{bmatrix} \begin{bmatrix}
i_a \\
i_b \\
i_c 
\end{bmatrix} + \psi_m \begin{bmatrix}
\cos(\theta_r) \\
\cos(\theta_r - \frac{2\pi}{3}) \\
\cos(\theta_r - \frac{4\pi}{3}) 
\end{bmatrix}
\]

- Flux produced by the rotor magnet

- Leakage inductance

- Magnetising inductance

\[
L_a = L_b = L_c = L_l + L_m
\]

- Self inductance

\[
M_{ab} = M_{bc} = M_{ca} = -\frac{L_m}{2}; \quad M_{ab} = M_{ba}; \quad M_{bc} = M_{cb}; \quad M_{ca} = M_{ac}
\]

- Mutual inductance
PMSM Dynamic Equations

- Voltage vector equation in the stationary $\alpha$-$\beta$ frame
  - Replacing the inductances values and applying the space vector transformation

\[
\begin{bmatrix}
v_\alpha \\
v_\beta \\
\end{bmatrix} = r_s \begin{bmatrix}
i_\alpha \\
i_\beta \\
\end{bmatrix} + \left( L_l + \frac{3}{2} L_m \right) \frac{d}{dt} \begin{bmatrix}
i_\alpha \\
i_\beta \\
\end{bmatrix} + \psi_m \frac{d}{dt} \begin{bmatrix}
\cos(\theta_r) \\
\cos(\theta_r - \frac{\pi}{2}) \\
\end{bmatrix}
\]
PMSM Dynamic Equations

- **Saliency**
  - Variation of the stator phase inductance as function of the rotor position.

\[
L_a = L_l + \bar{L}_m - \Delta L_m \cos (2\theta_r)
\]

\[
L_b = L_l + \bar{L}_m - \Delta L_m \cos (2\theta_r - \frac{4\pi}{3})
\]

\[
L_a = L_l + \bar{L}_m - \Delta L_m \cos (2\theta_r - \frac{2\pi}{3})
\]

\[
M_{ab} = \frac{-L_m}{2} - \Delta L_m \cos (2\theta_r - \frac{2\pi}{3})
\]

\[
M_{bc} = \frac{-L_m}{2} - \Delta L_m \cos (2\theta_r - \frac{4\pi}{3})
\]

\[
M_{ca} = \frac{-L_m}{2} - \Delta L_m \cos (2\theta_r - \frac{2\pi}{3})
\]

For example
PMSM Dynamic Equations

- voltage vector equation in the stationary $\alpha$-$\beta$ frame considering saliency

\[
\begin{bmatrix}
    v_\alpha \\
    v_\beta
\end{bmatrix} = R_s \begin{bmatrix}
    i_\alpha \\
    i_\beta
\end{bmatrix} + \frac{d}{dt} \begin{bmatrix}
    L_s - \Delta L_s \cos(2\theta_r) & \Delta L_s \sin(2\theta_r) \\
    \Delta L_s \sin(2\theta_r) & L_s + \Delta L_s \cos(2\theta_r)
\end{bmatrix} \cdot \begin{bmatrix}
    i_\alpha \\
    i_\beta
\end{bmatrix} + \psi_m \frac{d}{dt} \begin{bmatrix}
    \cos(\theta_r) \\
    \cos(\theta_r - \frac{\pi}{2})
\end{bmatrix}
\]

- Where

\[
L_s = L_i + \frac{3}{2} L_m
\]

\[
\Delta L_s = \frac{3}{2} \Delta L_m
\]

- If $\Delta L_m = 0$ there is no saliency and it is obtained the previous equation.
PMSM Dynamic Equations

- Voltage vector equation in the synchronous reference d/q frame fixed on the rotor
  - Angle chosen equal to the PM position

\[
\begin{bmatrix}
    v_d \\
    v_q
\end{bmatrix} = r_s \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} L_d p & - L_q \omega_r \\ L_d \omega_r & L_q p \end{bmatrix} \cdot \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \psi_m \omega_r \begin{bmatrix} 0 \\ 1 \end{bmatrix}
\]

differential operator \( p \); direct axis \( L_d = L_s - \Delta L_s \) and quadrature axis inductances \( L_q = L_s + \Delta L_s \)
PMSM Dynamic Equations

- expression for the instantaneous torque for the PM synchronous machine

\[
T_e = \frac{3P}{2} \left\{ \psi_m i_q + i_d i_q (L_d - L_q) \right\}
\]

- first term, usually called as magnet torque, is directly proportional to \( i_q \) and independent of \( i_d \).
- second term, or reluctance torque, is only present in salient machines where \( L_d = 0 \) and is proportional to the current product \( i_d i_q \).

- Motion equation:
  - J rotor inertia.
  - D friction
Field Oriented Control of PMSM
FOC in PMSM

- instantaneous torque for the PM synchronous machine

\[ T_e = \frac{3P}{2} \{ \psi_m i_q + i_d i_q (L_d - L_q) \} \]

- \( i_d \) will be kept to zero, for not demagnetizing the PM machine. Therefore, the reluctance torque will be zero.
- Electromagnetic torque will be regulated with \( i_q \).

\[ T_e > 0 \]

\[ T_e < 0 \]
FOC Scheme

- 3 PI control loops
  - 2 identical inner current loops, $d$ and $q$ axis.
  - 1 outer speed loop.
Current PI Control

• Inner faster loop.
• D and Q current loops are closed by identical PI.
• From

\[
\begin{bmatrix}
    v_d \\
v_q
\end{bmatrix} = r_s \begin{bmatrix}
i_d \\
i_q
\end{bmatrix} + \begin{bmatrix}
L_d p & -L_q \omega_r \\
L_d \omega_r & L_q p
\end{bmatrix} \cdot \begin{bmatrix}
i_d \\
i_q
\end{bmatrix} + \psi_m \omega_r \begin{bmatrix} 0 \\
1
\end{bmatrix}
\]

• Eliminating d-q coupling terms

\[
\frac{i_{dq}(s)}{v_{dq}(s)} = \frac{1}{L_{dq} s + r_s}
\]
Current PI Control

Manufacturer’s data for the PM machine from Control Techniques under the commercial name UNIMOTOR.

<table>
<thead>
<tr>
<th>Model:</th>
<th>142UMC30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of poles:</td>
<td>6</td>
</tr>
<tr>
<td>Rated speed:</td>
<td>3000 (rpm)</td>
</tr>
<tr>
<td>Rated torque:</td>
<td>12.2 (Nm)</td>
</tr>
<tr>
<td>Rated power</td>
<td>3.82 (kW)</td>
</tr>
<tr>
<td>Kt:</td>
<td>1.6 (Nm/Arms)</td>
</tr>
<tr>
<td>Ke:</td>
<td>98.0 (Vrms/krpm):</td>
</tr>
<tr>
<td>Inertia:</td>
<td>20.5 (kgcm²)</td>
</tr>
<tr>
<td>R (ph-ph):</td>
<td>0.94 (Ohms)</td>
</tr>
<tr>
<td>L (ph-ph):</td>
<td>8.3 (mH)</td>
</tr>
<tr>
<td>Continuous stall:</td>
<td>15.3 (Nm)</td>
</tr>
<tr>
<td>Peak:</td>
<td>45.9 (Nm)</td>
</tr>
</tbody>
</table>

\[
\frac{i_{dq}(s)}{v_{dq}(s)} = \frac{1}{L_{dq} s + r_s}
\]

\[
\frac{i_{dq}(s)}{v_{dq}(s)} = \frac{1}{4.15 \cdot 10^{-3} s + 0.47}
\]
Current PI Control

- The plant can be simplified as follows
  - First order with one pole: $s = -\frac{r_s}{L_{dq}}$
  - PI transfer function: $PI(s) = K_p \frac{K_i}{s} = \frac{K_p s + K_i}{s}$

![Diagram of PI Control System]
Current PI Control

- Root locus and step response with a P controller
  - $E_0$. Position Error
  - Slow dynamics.

\[ K_p = 1; \quad K_I = 0 \]

\[ s = -\frac{r_s}{L_{dq}} \]

- Solution: add a PI controller
  - Add a zero and a pole

\[ PI(s) \frac{K_p s + K_I}{s}; \]

pole: \( s = 0 \)

zero: \( s = -\frac{K_I}{K_p} \)
Current PI Control

- Root locus and step response with a PI controller
  - $E_0 = 0$
  - 2nd order system and response
  - Damping factor equal to 0.707

\[ K_p = 5.5; \quad \frac{K_i}{K_p} = -800 \]
Current PI Control

- Implementing PI in a DSP
  - From S to Z domain
    \[ PI(s) = K_p \frac{s + \frac{k_i}{k_p}}{s} \quad \rightarrow \quad PI(z) = K_p \frac{z - (1 - \frac{k_i}{k_p}Ts)}{z - 1} \]
  - \(Ts=100\text{us}\)
    \[ PI(s) = 5,5 \frac{s + 800}{s} \quad \rightarrow \quad PI(z) = 5,5 \frac{z - 0,92}{z - 1} \]
Current PI Control

- Implementing a PI Controller in a DSP
  - From Z to discrete time domain
    \[ PI(z) = \frac{vq\_ref(z)}{iq\_error(z)} = K_p \frac{z - (1 - \frac{K_i}{K_p}Ts)}{z - 1} \]

    \[ vq\_ref(z)(z - 1) = iq\_error(z)[K_p(z - (1 - \frac{K_i}{K_p}Ts))] \]
    \[ vq\_ref(z)(1 - z^{-1}) = iq\_error(z)[K_p(1 - z^{-1}(c\_K_i\_K_p))] \]
    \[ vq\_ref = vq\_ref\_last + K_p(iq\_error - iq\_error\_last(c\_K_i\_K_p)) \]

  - C code for the TI DSP 6711
    ```c
    // start iq PI controller
    iq\_error = iq\_ref - iq;
    vq\_ref = vq\_ref\_last + c\_Kp \times (iq\_error - c\_Ki\_Kp \times iq\_error\_last);
    iq\_error\_last = iq\_error;
    
    if (vq\_ref > VPI\_MAX) vq\_ref = VPI\_MAX;
    if (vq\_ref < -VPI\_MAX) vq\_ref = -VPI\_MAX;
    vq\_ref\_last = vq\_ref;
    // end iq PI controller
    ```
Speed PI Control

- The plant can be simplified as follows
  - First order with one pole \( s = -\frac{D}{J} \)
  - Mechanical time constant might be 50 times slower than the electrical one. Current loop is neglected.
  - Typical sampling time 5ms.

\[
\omega^* \quad + \quad - \quad \text{PI} \quad \frac{1}{J s + D}
\]
Sensorless FOC by High Frequency Injection
General Scheme

HF Injection Vector

\[ \begin{bmatrix} v_{\alpha i} \\ v_{\beta i} \end{bmatrix} = \hat{V}_i \begin{bmatrix} -\sin(\omega_i t) \\ \cos(\omega_i t) \end{bmatrix} \]

\[ \omega_i = 1 \text{ kHz} \]

Compensated Position Signals

Homodyne Signal Processing
Dynamic HF PMSM model

• An AC machine is said to be salient if \( L_d \neq L_q \)

• In SMPMPSM, the geometric saliency is very small. Therefore, it is track the saturation saliency.

\[
\begin{align*}
\begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} &= \begin{bmatrix} r_s & 0 \\ 0 & r_s \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} + \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} \begin{bmatrix} \lambda_\alpha \\ \lambda_\beta \end{bmatrix} \\
\begin{bmatrix} \lambda_\alpha \\ \lambda_\beta \end{bmatrix} &= \begin{bmatrix} \bar{L}_s - \Delta L_s \cos(2\theta_r) & -\Delta L_s \sin(2\theta_r) \\ -\Delta L_s \sin(2\theta_r) & \bar{L}_s + \Delta L_s \cos(2\theta_r) \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} + \lambda_m \begin{bmatrix} \cos(\theta_r) \\ \sin(\theta_r) \end{bmatrix}
\end{align*}
\]

Where: \( \bar{L}_s = \frac{L_q + L_d}{2} \); \( \Delta L_s = \frac{L_q - L_d}{2} \)
Dynamic HF PMSM model

- Injecting a rotating HF voltage vector:
  \[ \mathbf{v}_i = \begin{bmatrix} v_{\alpha i} \\ v_{\beta i} \end{bmatrix} = \hat{V}_i \begin{bmatrix} -\sin(\omega_i t) \\ \cos(\omega_i t) \end{bmatrix} \]

- The following HF current is obtained:
  \[ \mathbf{i}_i = \begin{bmatrix} i_{\alpha i} \\ i_{\beta i} \end{bmatrix} = \begin{bmatrix} I_0 \cos(\omega_i t) + I_1 \cos(2\theta_r - \omega_i t) \\ I_0 \sin(\omega_i t) + I_1 \sin(2\theta_r - \omega_i t) \end{bmatrix} \]

- The amplitude of the negative sequence \( I_1 \) is proportional to the saliency \( \Delta L = (L_d - L_q)/2 \):
  \[ I_0 = \frac{\hat{V}_i \bar{L}}{L_d L_q \omega_i} ; \quad I_1 = \frac{\hat{V}_i \Delta L}{L_d L_q \omega_i} \]

- Frequency domain representation:
Homodyne Signal Processing

- Frequency domain
Homodyne Signal Processing

- Time domain
Improving the position signals

- Harmonics exist on resolver signals
  - Non-sinusoidal distribution of saturation
  - Inverter effects – dead time & device voltage drop

Cleaning process
SMP Tables
Experimental results

4kW SMPM machine sensorless Position Control – 0% load

- Response to 180° position demand
Experimental results

4kW SMPM machine sensorless Position Control – 100% load

- Response to 180° position demand
  - no integrator in control loop (incremental position only)
  - $i_{sq}$ (torque current) limited to 1.3 x rated
Sensorless Control with Matrix Converter

Experimental Set up

- **Matrix Converter**
  - 7.5kW
- **Surface Mount Permanent Magnet Motor**
  - 4kW

- Basics for DSP implementation
- C:\MATLAB7\work\foc
- File: foc.mdl
Conclusions

- Permanent Magnet Synchronous Motors
  - Features / Advantages
    - No rotor currents => no rotor losses.
    - Higher efficiency => energy saving capability.
    - Smaller rotor diameters, higher power density and lower rotor inertia.
    - Weight and volume less than other type of machine for the same power. Attractive for aerospace applications such as aircraft actuators.
    - Other applications: machine tools, position servomotors (replacing the DC motors).
  - Inconvenience:
    - synchronous machines => need for rotor position.
    - Price
Conclusions

- PMSM motor model has been obtained
- Field Oriented Control for PMSM has been presented
  - PI control loops
- HF injection Sensorless vector control has been introduced.
  - Scheme and principles.
  - Saturation saliency has been tracked to estimate the rotor position.
  - 4 step homodyne demodulation.
  - Position signal improvements:
    - Dead time compensation.
    - SMP.
Questions / Debate

Moltes gràcies

Direct Torque Control
- the world's most advanced AC drive technology

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